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Generalized gyrotron theory with inclusion of adiabatic electron trapping in the presence of a depressed collector

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The effect of electrostatic electron trapping in the presence of a depressed collector has been included in gyrotron efficiency computations. The results are presented as general contour plots in the control parameter plane. A condition for trapping is derived in terms of the retarding voltage. The overall electron efficiency as a function of the retarding voltage has been calculated for a representative set of control parameters including the effect of electron velocity spread. It is found that the onset of trapping seriously decreases the efficiency. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1350667]

I. INTRODUCTION

In a gyrotron with a depressed collector there is a risk for electrons to be reflected back towards the resonator by the retarding potential. This reflection takes place whenever the height of the potential barrier exceeds the longitudinal kinetic energy of an electron in the collector region. In Ref. 1 it was found via simulation that the electrons are eventually swept out to the collector by the high-frequency field, and no significant charge accumulation would thus result. However, how the reflected electrons affect the gyrotron efficiency has not been reported.

Electron reflection by the retarding electrostatic potential can be regarded as an adiabatic process, if it takes place over a short distance compared to the Larmor period of the electron, that is, if $\Delta z_{\text{ret}} \ll z_{\text{L}}$ as stated in Ref. 2. Here Δz_{ret} is the spatial extension of the retarding region, and $z_{\text{L}} = p_{\|}/eB_{\text{coll}}$ is the Larmor period of an electron with a momentum component $p_{\|}$ in the direction of the magnetic field (B_{coll}) in the collector region. In this adiabatic case the reflected electron is expected to pass the resonator backwards, to reflect from the negative potential of the gun and to interact with the RF field again, following closely the magnetic field lines on its way. This process continues until the electron's energy at the cavity end suffices to carry it to the collector.

II. GYROTRON EFFICIENCY

The evolution of the transverse momentum of the electron p in a gyrotron resonator is governed by the equation

$$\frac{dp}{d\zeta} + i(\Delta + |p|^2 - 1)p = if(\zeta)F, \tag{1}$$

when operation at the first harmonic is considered. The initial condition is $p(\zeta_0) = \exp(i\vartheta_0)$ with $0 \le \vartheta_0 \le 2\pi$. Here p is normalized to its initial absolute value, ζ is the dimensionless axial coordinate, Δ is the frequency mismatch, $f(\zeta)$ is the high-frequency field profile in the resonator, and F is the coupling factor between the beam and the field. This description is the so-called cold-cavity approximation, in which f does not depend on the electron motion but only on the geometry of the resonator. In this case the field is well approximated by a Gaussian:

$$f(\zeta) = \exp\left[-\left(\frac{2\zeta}{\mu} - \sqrt{3}\right)^2\right], \tag{2}$$

the parameter μ being the dimensionless length of the resonator. Equation (1) is to be solved from $\zeta_0 = 0$ to $\zeta_{\text{out}} = \sqrt{3}\mu$.

For a detailed description of the quantities p, ζ, Δ, F, and μ, see e. g.[3–5]. The electron perpendicular efficiency $\eta_\perp$ can be calculated using the solutions of (1) by means of the expression

$$\eta_\perp = 1 - \frac{1}{2\pi}\int_0^{2\pi} |p(\zeta_{\text{out}})|^2 \, d\vartheta_0. \tag{3}$$

From this, the total electron efficiency is obtained by

$$\eta_{\text{el}} = \frac{\alpha^2}{1+\alpha^2}\eta_\perp, \tag{4}$$

where $\alpha = \beta_{\perp 0}/\beta_{\|0}$ is the pitch factor and $\beta_{\perp 0}$ and $\beta_{\|0}$ are dimensionless transverse and longitudinal velocities of the electron at the entrance to the cavity. Using a depressed collector at the potential U_{coll} one can increase the efficiency by the factor $U_{\text{cath}}/(U_{\text{cath}} - U_{\text{coll}})$, where U_{cath} is the accelerating voltage.

III. VELOCITY SPREAD

We handle the transverse velocity spread of electrons in line with Ref. 6. The quantities μ, F, and Δ can be expressed as

$$\mu = C_\mu^\beta \frac{\beta_\perp^2}{\beta_\|}, \tag{5}$$

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$$F = \frac{C_F^\beta}{\beta_\perp^3}, \tag{6}$$

$$\Delta = \frac{C_\Delta^\beta}{\beta_\perp^2}, \tag{7}$$

where C_μ^β, C_F^β and C_Δ^β do not depend on $\beta_\perp$. The longitudinal velocity $\beta_\parallel$ can be written using the perpendicular velocity $\beta_\perp$ and the relativistic factor γ of the electrons as $\beta_\parallel = (1 - 1/\gamma^2 - \beta_\perp^2)^{1/2}$. Introducing the error ϵ of the transverse velocity we find that

$$\mu_\epsilon^\beta = C_\mu^\beta \frac{\beta_{\perp 0}^2(1+\epsilon)^2}{\sqrt{1 - 1/\gamma^2 - \beta_{\perp 0}^2(1+\epsilon)^2}}, \tag{8}$$

$$F_\epsilon^\beta = \frac{C_F^\beta}{\beta_{\perp 0}^3(1+\epsilon)^3}, \tag{9}$$

$$\Delta_\epsilon^\beta = \frac{C_\Delta^\beta}{\beta_{\perp 0}^2(1+\epsilon)^2}, \tag{10}$$

$$\alpha_\epsilon^\beta = \frac{\beta_{\perp 0}(1+\epsilon)}{\sqrt{1 - 1/\gamma^2 - \beta_{\perp 0}^2(1+\epsilon)^2}}. \tag{11}$$

These can be written as

$$\mu_\epsilon^\beta = \frac{\mu(1+\epsilon)^2}{\sqrt{1 - 2\alpha^2\epsilon - \alpha^2\epsilon^2}}, \tag{12}$$

$$F_\epsilon^\beta = \frac{F}{(1+\epsilon)^3}, \tag{13}$$

$$\Delta_\epsilon^\beta = \frac{\Delta}{(1+\epsilon)^2}, \tag{14}$$

$$\alpha_\epsilon^\beta = \frac{\alpha(1+\epsilon)}{\sqrt{1 - 2\alpha^2\epsilon - \alpha^2\epsilon^2}}. \tag{15}$$

IV. CONDITION FOR ELECTRON TRAPPING

To connect the retarding potential with the dimensionless momentum $p(\zeta_{\text{out}})$ obtained by solving (1) numerically, recall that all momenta are normalized to the initial value of transverse momentum: $p = \mathcal{P}_\perp/\mathcal{P}_{\perp 0}$ and $p_\parallel = \mathcal{P}_\parallel/\mathcal{P}_{\perp 0}$. Using the relation between the total energy of an electron and the accelerating voltage U_{cath},

$$(\mathcal{P}_{\perp 0}^2 + \mathcal{P}_{\parallel 0}^2)c^2 + (m_e c^2)^2 = W_{\text{tot}}^2 = (m_e c^2 + eU_{\text{cath}})^2,$$

and equations for the momentum components as a function of normalized velocity,

$$\mathcal{P}_{\perp 0} = \beta_{\perp 0}\gamma m_e c \quad \text{and} \quad \mathcal{P}_{\parallel 0} = \beta_{\parallel 0}\gamma m_e c$$

together with $\gamma = (1 - {\beta_{\parallel 0}}^2 - {\beta_{\perp 0}}^2)^{-1/2}$ and the definition of the pitch factor $\alpha = \beta_{\perp 0}/\beta_{\parallel 0}$ we find that

$$\mathcal{P}_{\perp 0} = \alpha m_e c \sqrt{\frac{2\phi_{\text{cath}} + \phi_{\text{cath}}^2}{1+\alpha^2}} \tag{16}$$

and

$$\mathcal{P}_{\parallel 0} = m_e c \sqrt{\frac{2\phi_{\text{cath}} + \phi_{\text{cath}}^2}{1+\alpha^2}}. \tag{17}$$

Potentials have been normalized as $\phi = eU/m_e c^2$.

Between the cavity end and the retarding potential hill the electron experiences a diverging magnetic field but no electric field. Its total kinetic energy is therefore conserved,

$$\mathcal{P}_\perp(z)^2 + \mathcal{P}_\parallel(z)^2 = \text{const}, \tag{18}$$

but the transverse motion slows down along with the weakening of the magnetic field[2]:

$$\frac{\mathcal{P}_\perp(z)^2}{B(z)} = \text{const}. \tag{19}$$

In the vicinity of the collector the magnetic field B_{coll} is weak and practically constant. Applying the conservation laws (18) and (19) at the cavity end ($z = z_{\text{out}}$, $B = B_{\text{cav}}$) and at the collector ($B = B_{\text{coll}}$), we obtain the expression for the axial kinetic energy of the electron at the collector

$$W_\parallel(z_{\text{coll}}) = \sqrt{\left[\mathcal{P}_\parallel(z_{\text{out}})^2 + \left(1 - \frac{B_{\text{coll}}}{B_{\text{cav}}}\right)\mathcal{P}_\perp(z_{\text{out}})^2\right]c^2 + (m_e c^2)^2} - m_e c^2,$$

which is available for surpassing the retarding potential U_{coll}. Those electrons, for which

$$W_\parallel(z_{\text{coll}}) > eU_{\text{coll}},$$

reach the collector. For the (complex) dimensionless transverse momentum $p(\zeta_{\text{out}}) = \mathcal{P}_\perp(z_{\text{out}})/\mathcal{P}_{\perp 0}$ this condition reads

$$|p(\zeta_{\text{out}})| > \frac{m_e c}{\mathcal{P}_{\perp 0}} \sqrt{\frac{2\phi_{\text{coll}} + \phi_{\text{coll}}^2 - \mathcal{P}_{\parallel 0}^2/m_e^2c^2}{1 - B_{\text{coll}}/B_{\text{cav}}}}.$$

Using (16) and (17), this critical value can be expressed as a function of the voltages, magnetic fields, and the pitch factor:

$$|p(\zeta_{\text{out}})| > \frac{1}{\alpha\sqrt{1-b}} \sqrt{\frac{2\phi_{\text{coll}} + \phi_{\text{coll}}^2}{2\phi_{\text{cath}} + \phi_{\text{cath}}^2}(1+\alpha^2) - 1} \equiv p_{\text{cut-off}}. \tag{20}$$

Here $b = B_{\text{coll}}/B_{\text{cav}}$. We call the right-hand side $p_{\text{cut-off}}$ since all electrons with $|p(\zeta_{\text{out}})| < p_{\text{cut-off}}$ are "cut off" from the spectrum by reflection. In the case $2\phi_{\text{coll}} + \phi_{\text{coll}}^2 < \mathcal{P}_{\parallel 0}^2/m_e^2c^2$ the initial longitudinal momentum alone suffices to bring the electron to the collector and we state that $p_{\text{cut-off}} = 0$.

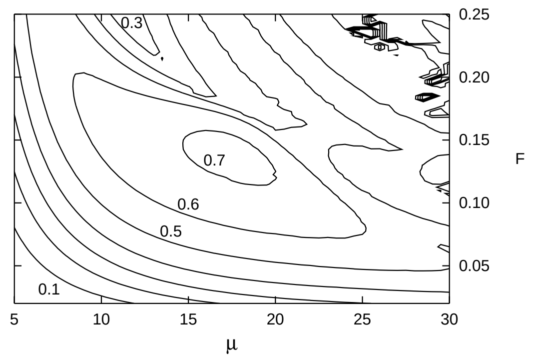

FIG. 1: The contours of the perpendicular efficiency $\eta_\perp$ with $p_{\text{cut-off}} = 0$.

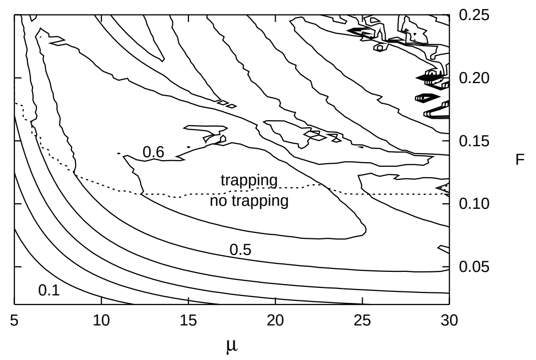

FIG. 2: The contours of the perpendicular efficiency $\eta_\perp$ with $p_{\text{cut-off}} = 0.2$. The dashed line is the boundary between the regions with and without trapping.

V. COMPUTATIONS

A. General efficiency plots

We model the process of successive interactions and reflections by integrating numerically the gyrotron equation (1) and repeating the integration until the electron reaches the collector. The motion of an electron traveling backwards in the resonator can be most conveniently found by performing the integration with the standard routine from $\zeta = \zeta_{\text{out}}$ to $\zeta = \zeta_0$ with a negative step size.

The time that the electrons spend outside the resonator between two successive interactions is long compared to their Larmor period. We therefore completely lose the information about the phase of rotation. The energy, instead, is conserved due to the assumption of adiabaticity. These facts are taken into account by keeping the absolute value of momentum fixed and randomizing the phase angle each time before starting a new integration.

The need for giving electrons a new, random phase arises from the fact that Eq. (1) is symmetric with respect to reversal of the direction of ζ. Namely, if $p_1(\zeta)$ is the unique solution of (1) corresponding to the initial value

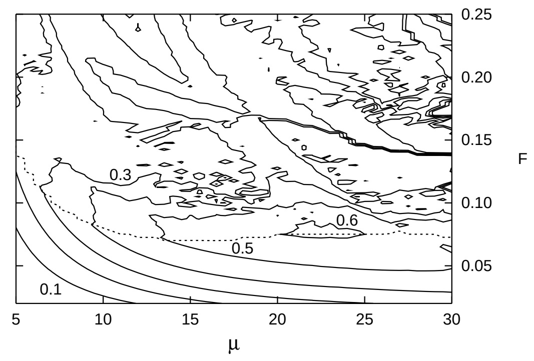

FIG. 3: The contours of the perpendicular efficiency $\eta_\perp$ with $p_{\text{cut-off}} = 0.4$. The dashed line is the boundary between the regions with trapping (above) and without trapping (below).

$p_1(0) = p_0$, and $p_1(\zeta_{\text{out}}) = p_{\text{out}}$, then $p_1'(\zeta') = p_1(\zeta)$ (where $\zeta' = \zeta_{\text{out}} - \zeta$) will satisfy the initial value problem

$$\frac{dp}{d\zeta'} + ip(\Delta + |p|^2 - 1) = iFf'(\zeta'), \quad p(\zeta' = 0) = p_{\text{out}}$$

and yield $p_1'(\zeta_{\text{out}}) = p_0$ again. Without phase randomizing the electrons would follow the same trajectory back and forth within the limits of numerical accuracy.

The results of the computations are presented in Figs. 1–3 as contour plots of perpendicular efficiency $\eta_\perp$ in the (μ, F)-plane. In Fig. 1 the standard plot without electron trapping is shown for comparison. The parameter Δ is set in each point to the optimum value with respect to the perpendicular efficiency when $p_{\text{cut-off}} = 0$. This value of Δ is then used for all other values of $p_{\text{cut-off}}$. In the examples of the efficiency plots with $p_{\text{cut-off}} \neq 0$ in Figs. 2 and 3 we also indicate the boundary of the region where trapping occurs.

B. Computations with velocity spread

To model the trapping effect more realistically, we chose three representative combinations of μ and F [($\mu = 10$, $F = 0.10$), ($\mu = 17$, $F = 0.125$), ($\mu = 25$, $F = 0.075$)] and computed the electron efficiency as a function of retarding voltage, simultaneously introducing a spread in the electron transverse velocity. This was done by applying (20) with (12)–(15). A Gaussian,

$$f_e(\beta_\perp) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\beta_\perp - \beta_{\perp 0})^2}{2\sigma^2}\right], \tag{21}$$

is a suitable approximation for the velocity distribution: see Ref. 7. The relation between the rms deviation σ and the velocity spread $\delta\beta_\perp$ can be written as

$$\delta\beta_\perp = 1.8\sqrt{2}\sigma, \tag{22}$$

where $\delta\beta_\perp$ is defined as

$$\delta\beta_\perp = \frac{1}{2}\left(\frac{\beta_{\perp,\text{max}} - \beta_{\perp,\text{min}}}{\beta_{\perp,\text{center}}}\right). \tag{23}$$

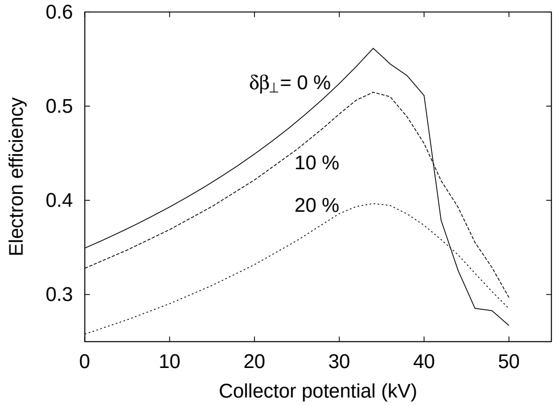

FIG. 4: The electron efficiency as a function of retarding potential at $\mu = 10.0$ and $F = 0.10$, calculated with statistically independent variables.

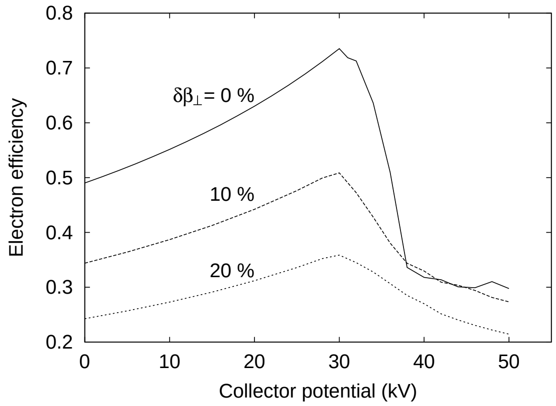

FIG. 5: The electron efficiency as a function of retarding potential at $\mu = 17.0$ and $F = 0.125$, calculated with statistically independent variables.

In addition to an ideal beam with zero spread we used $\delta\beta_\perp = 10\,\%$ and $20\,\%$. The corresponding values of σ are 0.0393 and 0.0786. From (12) it follows that $|\epsilon|$ must be smaller than $\epsilon_{\rm max} = (1 + 1/\alpha^2)^{1/2}$, which is related to the fact that the physical boundaries of $\beta_\perp$ are $0 \le \beta_\perp \le (1-1/\gamma^2)^{1/2}$. In practice the efficiency was calculated by varying ϵ between $-0.95\,\epsilon_{\rm max}$ and $+0.95\,\epsilon_{\rm max}$ and averaging the corresponding efficiencies with a properly normalized weight function of the form (21). This method is an equivalent alternative for the Monte Carlo approach used in Ref. 6. However, it suffers less from random fluctuations with equal computational effort.

As in Ref. 6, the quantities μ_ϵ^β, F_ϵ^β, and Δ_ϵ^β were treated as statistically independent variables assigning a different ϵ to each of them. This independence of errors for the three quantities μ, F, and Δ simulates the fact that in real gyrotrons a change of $\beta_\perp$ always influences other gyrotron operating parameters. For example, the field profile (the effective length of the cavity), the quality factor of the cavity, and the operation frequency depend on the properties of the electron beam,

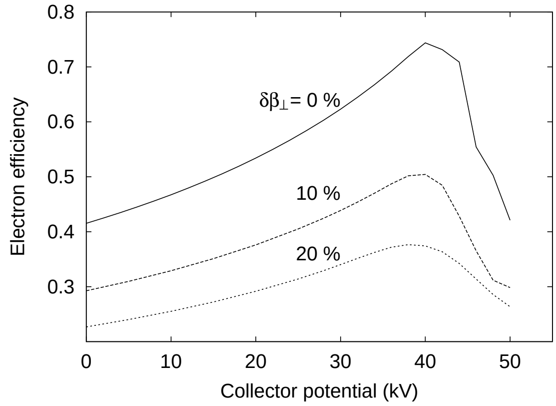

FIG. 6: The electron efficiency as a function of retarding potential at $\mu = 25.0$ and $F = 0.075$, calculated with statistically independent variables.

which means that the parameters μ, F, and Δ actually are complicated functions of $\beta_\perp$. Quantitatively this can be taken into account only in self-consistent calculations, which is beyond the scope of the present theory. The results of the present calculations are shown in Figs. 4–6.

However, if we make the most conservative but less realistic assumption that the variation of $\beta_\perp$ does not affect any other quantity, we should use one and the same ϵ in calculating μ_ϵ^β, F_ϵ^β, Δ_ϵ^β, and α_ϵ^β according to Eqs. (12)–(15). In this approach, which we call statistically dependent, we also calculated $p_{\text{cut-off}}$ from (20) for each $\beta_\perp$ using the corrected α of Eq. (15). The results of such calculations are presented in Figs. 7–9. It is obvious that here the efficiency deterioration due to velocity spread is much smaller than in the former case.

All computations have been performed for $\alpha = 1.5$ and $U_{\rm cath} = 90\,{\rm kV}$.

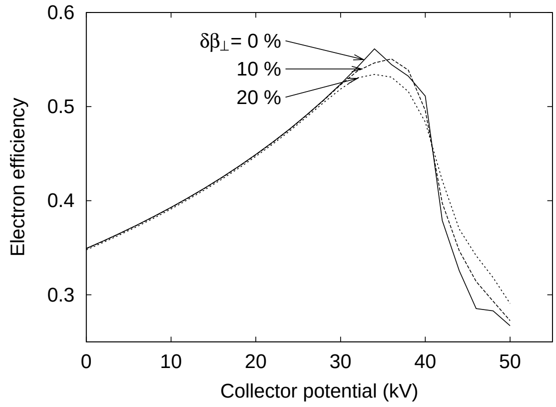

FIG. 7: Same as Fig. 4, but with statistically dependent variables.

VI. CONCLUSIONS

A formalism has been developed for inclusion of the effect of electrostatic trapping of electrons in the presence

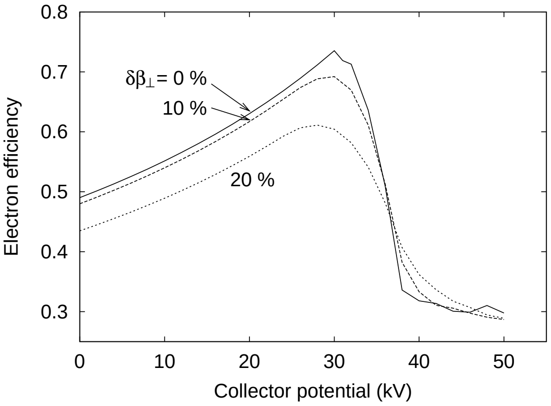

FIG. 8: Same as Fig. 5, but with statistically dependent variables.

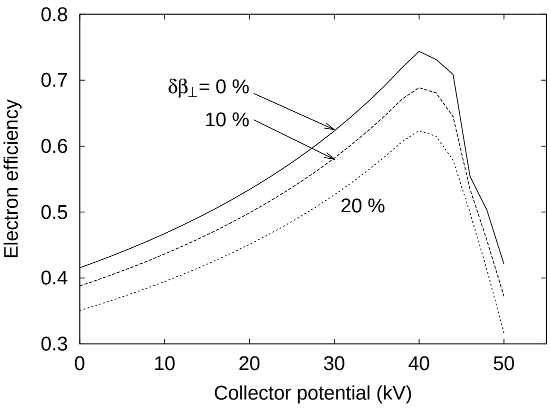

FIG. 9: Same as Fig. 6, but with statistically dependent variables.

of a depressed collector into gyrotron efficiency calculations. Several representative sets of gyrotron parameters have been chosen as examples.

It is evident from Figs. 2 and 3 that the trapping effect is more probable and pronounced in the operating regimes with high perpendicular efficiency. Indeed, here many electrons have very low rest energies at the exit from the cavity. They make the largest contributions to efficiency. At the same time these "good" electrons have the strongest tendency to become trapped. For example, at the point of the highest efficiency ($\mu = 17$, $F = 0.125$) with $U_{\rm coll} = 33\,{\rm kV}$ some of the "best" electrons pass the cavity six times before they acquire the energy from the RF field which is needed for leaving the trap. It is also seen (Figs. 5 and 8) that just for these μ and F values the increase of the efficiency due to the retarding potential as $U_{\rm cath}/(U_{\rm cath} - U_{\rm coll})$ breaks down already at $\sim 30\,{\rm kV}$ because of the onset of trapping, while in two other cases trapping begins to manifest itself at higher collector voltages. It is also observed that generally the velocity spread tends to smooth the transition to the trapping region.

In gyrotrons whose cavities have a low quality factor the dependences shown in Figs. 4–6 should be observed, while in the case of high quality factors weaker dependence on the velocity spread such as shown in Figs. 7–9 should be expected.

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