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Degradation of operation mode purity in a gyrotron with an off-axis electron beam

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The two-dimensional self-consistent time-dependent theory of beam-wave interaction in gyrotron resonators has been modified to account for eccentricity of the annular hollow electron beam. Numerical analysis yields the effect of beam eccentricity on the previously known limiting values of the azimuthal index of the mode, beyond which stationary single-mode operation becomes impossible. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1528938]

I. INTRODUCTION

High-power high-frequency gyrotrons play an essential role as microwave sources for plasma heating and current drive in all modern magnetic fusion machines. As the size and performance of experimental devices is increased towards a commercial reactor, also the requirements for frequency and unit output power of gyrotrons tighten. It is therefore important to know the operational limits of such gyrotrons. One significant group of limitations may arise from stochastic processes, whose onset becomes more likely with increasing output power. Stochastic behavior can be seen both in electron trajectories and in rf oscillations.

In Ref. 1 it was proved in cold-cavity approximation that under the influence of a high-frequency field with a Gaussian-type axial profile, possible chaos in electron trajectories can be only transient. After leaving the interaction cavity the electrons will again follow regular trajectories. In numerical studies with a realistic field[2] we have found that electron trajectories in a gyrotron resonator can become stochastic in the vicinity of some particular initial conditions. Under normal operating conditions, that is, in the regimes of high efficiency, there are few such trajectories and they do not significantly disturb power generation or the operation of depressed collectors.

As far as stochasticity of rf oscillations is concerned, detailed analyses of the one-dimensional self-consistent theory of gyrotron oscillations (see Refs. 3–5) have shown that gyrotrons can generate nonstationary oscillations in addition to stationary ones. In fact, it seems to be an intrinsic property of gyrotrons that the output signal is periodically modulated or even chaotic for certain operating conditions. The regimes of different types of oscillations form a complicated map in the operating parameter plane[6]. These findings are supported by recent experimental results[7]. However, for high-power high-frequency gyrotrons it is relevant to adopt a two-dimensional model described in Refs. 8 and 9 due to their large-diameter interaction cavities and short wavelengths. Such a model discards the representation of the rf field as TE modes and corresponding Bessel functions and, instead, treats the envelope of the field as a function of both the axial and the azimuthal coordinate. In Ref. 10 it was shown that stationary single-mode operation of a gyrotron becomes impossible above certain values of the azimuthal index m of the mode.

In practice several factors may make the electron beam deviate transversally from its desired position in the cavity. The shorter the wavelength, the more sensitive the tube is to beam misalignment. The effects of misalignment on efficiency, starting current, mode interaction, and frequency shift have been reported earlier (see, for example, Refs. 11–14 and references therein). In this paper I report two-dimensional self-consistent calculations of the temporal behavior of gyrotron oscillations in the case of a misaligned electron beam. The results obtained in Ref. 10 are generalized by relaxing the requirement of a concentrically placed beam. Some reasonable assumptions have been made to limit the number of new parameters. The effect of beam eccentricity on the oscillations was computed for two combinations of the generalized gyrotron variables Δ (cyclotron resonance mismatch) and I (dimensionless current).

II. THEORY

In the ideal case of a concentrically placed electron beam the electron motion and the time- and spatially-dependent high-frequency field in the resonator can be calculated from[9]

$$\begin{cases} \dfrac{dp}{d\zeta} + i(\Delta + |p|^2 - 1)p = if(\zeta,\xi,\tau), \\ \dfrac{\partial^2 f}{\partial \zeta^2} - i\dfrac{\partial f}{\partial \tau} - i\dfrac{\partial f}{\partial \xi} + \delta f = \dfrac{I}{2\pi}\displaystyle\int_0^{2\pi} p\, d\vartheta_0, \end{cases} \tag{1}$$

where p is the complex transverse momentum of the electron normalized to its initial absolute value, $\zeta = (\beta_{\perp 0}^2 \omega / 2\beta_{\parallel 0} c) z$ and $\xi = \frac{1}{8}\alpha^2 \beta_{\perp 0}^2 m\varphi$ are dimensionless axial and azimuthal coordinates, respectively, $\beta_{\perp 0} = v_{\perp 0}/c$ and $\beta_{\parallel 0} = v_{\parallel 0}/c$ are normalized electron velocities, $\alpha = \beta_{\perp 0}/\beta_{\parallel 0}$ is the pitch factor, $\Delta = 2(\omega - \omega_c)/\beta_{\perp 0}^2 \omega$ is the frequency mismatch, $\omega_c/2\pi = 28B/\gamma_{rel}$ is the electron cyclotron frequency in GHz, B is the magnetic field in T, γ_{rel} is the relativistic factor of electrons, $f(\zeta,\xi,\tau)$ is the high-frequency field in the resonator, $\tau = \frac{1}{8}\beta_{\perp 0}^4 \beta_{\parallel 0}^{-2} \omega_c t$ is the dimensionless time, $\delta = 8\beta_{\parallel 0}^2 \beta_{\perp 0}^{-4} [\varpi - \omega(\zeta)]{\omega_c}^{-1}$ describes variation of the cut-off frequency $\omega(\zeta)$ along the resonator axis, $\overline{\omega}$ is the cut-off frequency at the exit from the resonator, and I is the dimensionless current:

$$I = 9.4 \cdot 10^{-4} I_0 \beta_{\parallel 0} \beta_{\perp 0}^{-6} \frac{J_{m\pm1}^2\left(\frac{2\pi}{\lambda}R_{\rm el}\right)}{\gamma_{\rm rel}(\nu^2 - m^2)J_m^2(\nu)}. \quad (2)$$

Here I_0 is the beam current in amperes, J is the Bessel function, m is the azimuthal index of the mode, λ is the wavelength, $R_{\rm el}$ is the electron beam radius, and ν is the zero of the derivative of the Bessel function. This description is valid for operation at the fundamental cyclotron resonance.

The system of equations (1) has to be supplemented by the standard initial condition for the momentum, $p(0) = \exp(i\vartheta_0)$ with $0 \le \vartheta_0 \le 2\pi$, and by the boundary condition for the field at the entrance to the interaction space:

$$f(0, \tau) = 0, \quad (3)$$

which means that at the entrance the field must vanish. At the exit from the interaction space ($\zeta = \zeta_{\rm out}$) the so-called reflectionless boundary condition is applied:

$$\left(f(\zeta,\xi,\tau)\frac{1}{\sqrt{\pi i}}\int_0^\tau \frac{1}{\sqrt{\tau-\tau'}}\frac{\partial f(\zeta,\xi,\tau')}{\partial\zeta}d\tau'\right)\Bigg|_{\zeta=\zeta_{\rm out}} = 0. \quad (4)$$

In the azimuthal direction periodic boundary conditions are used. Finally, an initial condition for the field is needed. A field profile with only one maximum in the axial direction and a small deviation from perfect symmetry in the azimuthal direction can be described by the initial condition

$$f(\zeta,\xi,0) = \left[0.1 + 0.01\sin\left(\frac{2\pi\xi}{\xi_{\rm max}}\right)\right]\sin\left(\frac{\pi\zeta}{\zeta_{\rm out}}\right), \quad (5)$$

which has been used in all computations of this study. Here $\xi_{\rm max} = \frac{\pi}{4}\alpha^2{\beta_{\perp 0}}^2 m$.

If the hollow beam is shifted with respect to the resonator in the transverse direction, the description above must be slightly modified. Assuming the beam thickness to be negligible, the ideal (dashed line) and shifted (solid) beams can be described by the circles in Fig. 1. Due to a shift D, the distance of guiding centers from the cavity axis has changed from $R_{\rm el}$ to $R(\varphi)$ at the azimuthal angle φ. These quantities are related as

$$R_{\rm el}^2 = R(\varphi)^2 + D^2 - 2R(\varphi)D\cos\varphi,$$

so that

$$R(\varphi) = D\cos\varphi + \sqrt{R_{\rm el}^2 - D^2\sin^2\varphi}. \quad (6)$$

The equations (1) can still be used to solve the evolution of the rf field. Only the current parameter I must be replaced by an azimuth-dependent parameter

$$\tilde{I}(\varphi) = \frac{J_{m\pm1}^2\left(\frac{2\pi}{\lambda}R(\varphi)\right)}{J_{m\pm1}^2\left(\frac{2\pi}{\lambda}R_{\rm el}\right)} \times I, \quad (7)$$

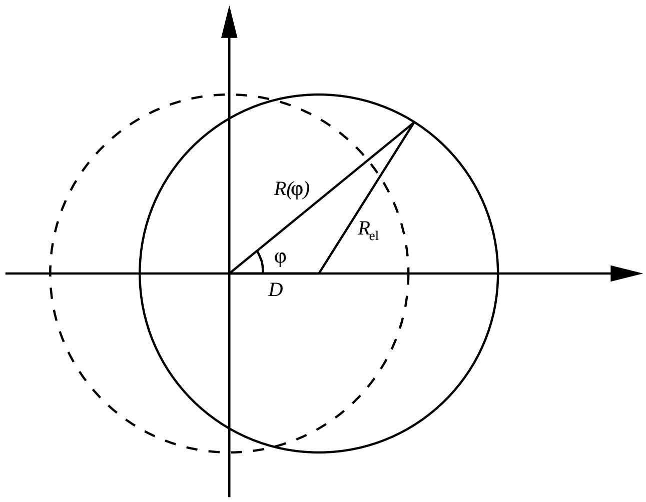

FIG. 1: Transversally by the distance d shifted electron beam (solid line) in the coordinate system of the resonator. The original position is indicated by the dashed circle.

where I should now be interpreted as the dimensionless current of the same beam if it were located concentrically with the resonator. Note that in many earlier papers on this subject (Refs. 11–14, for example) one proceeds by using the Graf's summation theorem for Bessel functions, which leads to an expression for $\tilde{I}$ averaged over φ. In this context, however, we need a parameter which is explicitly dependent on φ, and this is what our derivation so far has yielded. Making still the assumption that the intended position of the beam is at the first maximum of the Bessel function $J_{m\pm1}$, that is, $2\pi R_{\rm el}/\lambda = \nu_{m\pm1,1}$, where $\nu_{m,p}$ denotes the pth zero of the derivative of J_m, and introducing a dimensionless misplacement parameter $\tilde{d} = D/R_{\rm el}$, we obtain

$$\tilde{I}(\varphi) = \frac{J_{m\pm1}^2\left(\nu_{m\pm1,1}\left[\tilde{d}\cos\varphi + (1-\tilde{d}^2\sin^2\varphi)^{1/2}\right]\right)}{J_{m\pm1}^2\left(\nu_{m\pm1,1}\right)} \times I. \quad (8)$$

This is easily applied in the numerical code used in Ref. 10. One also needs to define the ranges of spatial coordinates. This was done by setting $\zeta_{\rm out} = 15$, $\beta_{\perp 0} = 0.426$, and $\beta_{\parallel 0} = 0.316$, which describe an "average" gyrotron resonator length, a typical accelerating voltage of 92 kV, and pitch factor $\alpha = 1.35$. While $\beta_{\perp 0}$ and $\beta_{\parallel 0}$ only scale the resulting $m_{\rm crit}$, one should be aware that qualitatively different results would be obtained with $\zeta_{\rm out}$ far from 15.

III. RESULTS

Numerical solution of (1) requires a lot of computation, and several runs are needed to determine the critical value of m with other parameters fixed. Therefore I have limited this study to two points in the parameter plane (Δ, I). The most significant point is the one giving maximum perpendicular efficiency in stationary single-mode operation, $(\Delta = 0.60, I = 0.01)$. Above $m_{\rm crit}$, the

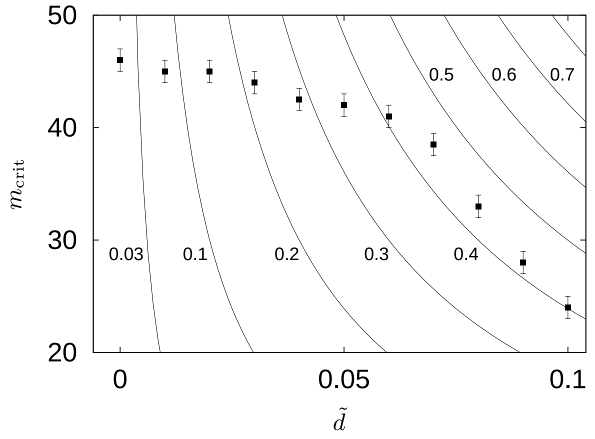

FIG. 2: Critical azimuthal index of the mode as a function of $\tilde{d} = D/R_{\rm el}$. Also the contours of $d = D/\lambda$ are shown. Here $\Delta = 0.60$, $I = 0.01$. The error bars indicate how precisely $m_{\rm crit}$ could be estimated with reasonable computational cost.

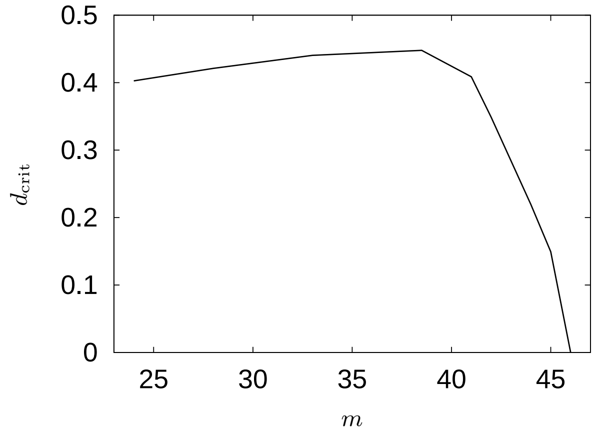

FIG. 4: Maximum tolerable displacement $d = D/\lambda$ as a function of the azimuthal index of the mode for $\Delta = 0.60$, $I = 0.01$.

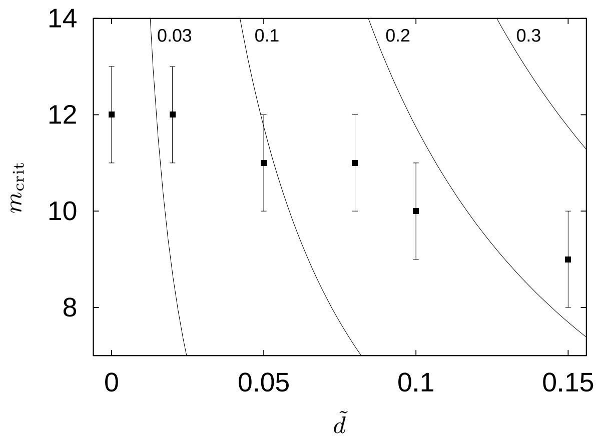

FIG. 3: Critical azimuthal index of the mode as a function of $\tilde{d} = D/R_{\rm el}$. Also the contours of $d = D/\lambda$ are shown. Here $\Delta = -0.60$, $I = 0.01$. The error bars indicate how precisely $m_{\rm crit}$ could be estimated with reasonable computational cost.

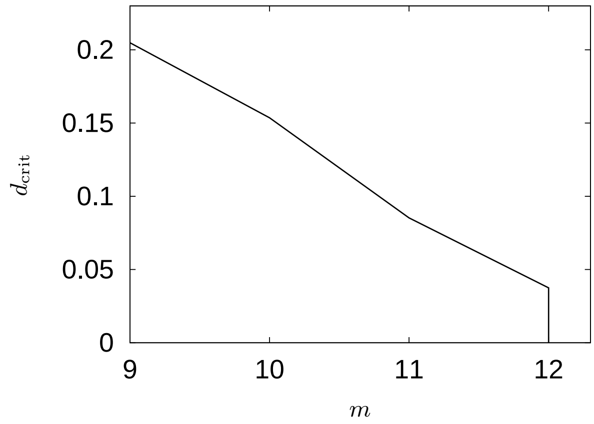

FIG. 5: Maximum tolerable displacement $d = D/\lambda$ as a function of the azimuthal index of the mode for $\Delta = -0.60$, $I = 0.01$.

field changes from its optimal shape into a less favorable configuration with two maxima in the axial direction. This transition is caused by competition between modes with different axial indices, and after a temporary coexistence the favorable mode is suppressed by its competitor giving lower efficiency, as described in Ref. 10. The other point of interest was chosen to lie symmetrically at $(\Delta = -0.60, I = 0.01)$ to make comparison between forward- and backward-wave interaction. It was found that the stability of backward-wave interaction is limited by gradually growing periodic oscillations when $m > m_{\rm crit}$. No completely chaotic oscillations due to beam misalignment were observed in the present study.

The variation of the critical value of the azimuthal index of the mode m is shown as a function of $\tilde{d} = D/R_{\rm el}$ in Figs. 2 and 3. While these data points serve well to extend our previous study[10] in terms of the formalism presented in this article, the same data are shown again in a more application-related way. Instead of $\tilde{d}$ one is often more interested in the quantity $d = D/\lambda$, whose contours are shown in Figs. 2 and 3. Plotting the maximum tolerable value $d_{\rm crit}$ of d as a function of m, we obtain Figs. 4 and 5.

IV. CONCLUSIONS

In this paper I have presented a straightforward way to include electron beam misalignment into the self-consistent two-dimensional model for time-dependent gyrotron oscillations. Due to the heavy computations required, the study was focused to two combinations of the generalized gyrotron parameters Δ and I. Both for positive and negative Δ, increasing misalignment tends to lower the threshold above which stationary single-mode operation becomes impossible. This is the effect one would expect symmetry-breaking to have. There exists also an opposite effect: the effective beam current decreases when the beam is shifted [see Eq. (7)], and more regular behavior is generally seen at lower currents. The results suggest, however, that symmetry breaking dominates in our cases.

From the practical point of view, the most significant result is shown in Fig. 4. It was previously known that with our choice of ζ_{out}, $\beta_{\perp 0}$, and $\beta_{\parallel 0}$, the ultimate limit for high-efficiency operation is at $m \approx 46$. Now it becomes evident that in the region $40 \lesssim m \lesssim 46$ the operating mode is very sensitive to beam misalignment. For example, the critical misplacement $d_{\mathrm{crit}} = 0.15$ for $m = 45$, which means 0.26 mm in a 170 GHz gyrotron. Together with high-order mode operation one should therefore always consider how precisely the beam can be placed into its correct position in the resonator. Once the wavelength and operating mode have been specified, Figs. 4 and 5 can be used for rough estimation of the tolerance in beam position.

For comparison with other effects of beam misalignment, Fig. 2 is useful. The slight decline of m_{crit} with increasing $\tilde{d}$ is accompanied by much more severe changes in efficiency and starting current. For example, when $\tilde{d} = 0.06$, m_{crit} is still about 40, but perpendicular efficiency has dropped by 32 % from its ideal value, and the starting current correspondingly increased by 47 % (see Refs. 11 and 14).

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