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Stability of A–B phase boundary in a constriction

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Abstract

We report on a Ginzburg–Landau calculation of the stability of a boundary between A and B phases of superfluid ^{3}He in a two-dimensional constriction. In the macroscopic limit the stability follows a well-known relation, which depends on the surface tension σ_{AB} of the A–B boundary. In the narrow-constriction limit the surface tension is not well defined, but the interface is always stable, and a weak link between the A and B phases is obtained.

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1. Pinning of A–B interface

Properties of the A–B interface in superfluid ^{3}He have previously been studied in some detail [1]. Most of the experimental and theoretical considerations were involved with nucleation problems, propagation of the free boundary, or determination of its order parameter structure and surface tension [2–4]. In contrast, we wish to study the conditions under which the boundary can be stabilized in a weak link, and the properties of the resulting current-phase characteristics. In this paper we concentrate on an analysis of the stability aspect, using the Ginzburg–Landau (GL) theory. We consider a two-dimensional model which approximates a long narrow slit in a planar wall.

It is well known that in the macroscopic limit the pinning stability of the interface is determined only by its finite surface tension σ_{AB} [2]. For a slit of width W the stability condition is

$$|\Delta f_{AB}| < \frac{2\sigma_{AB}}{W}, \qquad (1)$$

where $\Delta f_{AB} = f_A - f_B$ is the difference in bulk condensation energy densities of the two phases. However, the macroscopic concept of surface tension is not well defined in the limit of small constrictions, and Eq. (1) is not necessarily valid then. In any case, σ_{AB} is defined only for $\Delta f_{AB} \approx 0$. In the GL regime, Δf_{AB} can be varied by applying different pressures or magnetic fields. In this paper, we choose the first of these methods, and use in our calculations the Sauls–Serene strong-coupling parameters [5].

Fig. 1 shows schematically where the A–B boundary stabilizes at different ambient pressures. At a coexistence point $(T,p) = (T_0,p_0)$ in temperature–pressure plane both bulk phases are equally stable and $\Delta f_{AB} = 0$ (in GL theory T_0 is arbitrary). However, A phase is more stable inside the channel, and the interface will settle to the B phase end (a). Upon increasing the pressure, the interface will bulge (b) and eventually go unstable [2]. If the pressure is decreased, the opposite (d) will happen, but first a (rapid) transition through (c) must occur. The situation is not symmetric, and, a priori, the critical $|\Delta f_{AB}|$ may be different in opposite directions. There may also be hysteresis associated with the paths (a–b–a) or (a–d–a).

2. Numerical results

Our computational method employs a standard minimization routine for the GL free energy functional,

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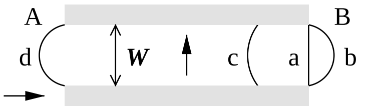

Fig. 1. Equilibrium position of the A–B interface in the channel at different pressures: (a) $p = p_0$, (b) $p > p_0$, (c,d) $p < p_0$. The thick arrows denote orientation of $\hat{l}$ vector when there is A phase in the respective regions.

which is discretized in and around the aperture. A lattice spacing of one ξ_{GL} was used in the larger apertures, where refinement had no noticeable effect on the stability behavior ($\xi_{GL} = \sqrt{K/\alpha}$ [4]). As boundary conditions, we assumed diffusive walls, and any orientations of the bulk A and B phase order parameters were made possible. Bulk cutoffs were chosen far enough, so that the phase boundary could freely bulge and escape at the instability.

For a free planar boundary, the lowest value of σ_{AB} should be attained when the bulk orientations of the A and B order parameters and the interface normal $\hat{s}$ satisfy $\hat{d}_\mu = \pm R_{\mu i}\hat{s}_i$ [1]. We shall restrict to this case here also, with $\hat{s}$ now the wall normal. Tests with other configurations show that the obtained critical values can fluctuate somewhat, sometimes causing the instability to occur slightly earlier.

We have mostly tested apertures in walls of two different thicknesses, $D/\xi_{GL} = 4$ or 20, and we found no difference in their critical $|\Delta f_{AB}|$ values (see Fig. 2). There is hardly any hysteresis in the B phase end (a–b–a), and the transition (a–c) is not clear-cut for channels this short. In contrast, there is strong hysteresis in the A phase end (a–d–a). This is because there is an energy barrier for A phase to re-enter the channel, since the inevitable bending of $\hat{l}$ requires the formation of a line singularity in one corner of the channel end. Hence, as the pressure is increased at point (d), the boundary tends to first freeze at the A phase end, and then jump directly to the B phase end.

From the asymptotic behavior of Fig. 2 we also conclude that, at $(T,p) = (T_0,p_0)$,

$$\sigma_{AB} = (0.72 \pm 0.2)\xi_T f_B, \quad (2)$$

where ξ_T is now defined as in Ref. [2] for simplicity of comparison. The upper and lower limits correspond to fits to the right and left branches, respectively. This result is in good agreement with the calculations of Refs. [3,4], although the macroscopic limit has only been approached up to $W/\xi_{GL} = 70$. This scheme for the determination of σ_{AB} is also closer in spirit to the experiment of Ref. [2], with the suppression of the order parameter at solid surfaces, the pinning, and the interfacial curvature all being taken into account.

As for the units of Fig. 2, note that f_A, f_B, and ξ_{GL} are themselves dependent on T and p. To be exact,

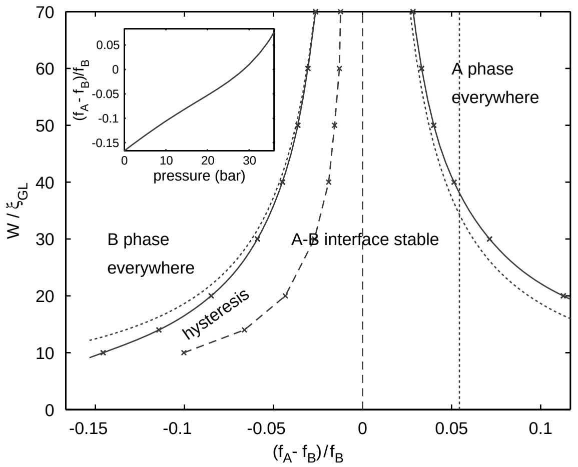

Fig. 2. Stability of the A–B interface inside a 2D slit. The solid curves calculated for different wall thicknesses (4 and 20) coincide. The dash–dotted line marks the hysteretic transition in the thick-wall case, where A phase penetrates back into the channel when the path (a–c–d) is reversed before the instability. The vertical dotted line corresponds to the melting pressure for the Sauls–Serene parameters [5], and the curved dotted lines to an extrapolation of Eqs. (1) and (2) for all p (see text).

the dotted curves corresponding to an extrapolation of Eqs. (1) and (2) assume that the path in (T,p) plane is taken such that $\xi_{GL}(T,p)f_B(T,p) = \xi_{GL}(T_0,p_0)f_B(T_0,p_0)$, with T_0 as a free parameter.

3. A–B interface and weak links

Ultimately, we are interested in the weak-link properties of the A–B interface. The most relevant regime in Fig. 2 is then $W/\xi_{GL} \leqslant 10$, since a significant suppression of the order parameter is required for a proper weak link realization [1]. In this case the pinned interface is stable for all physically obtainable pressures, and therefore a slit-like A–B weak link is well defined and experimentally feasible. As an interesting feature, we note that the associated current-phase relations are π periodic under the same condition $\hat{d}_\mu = \pm R_{\mu i}\hat{s}_i$ which was assumed above. These issues will be addressed elsewhere in more detail.

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